Probing the Brans-Dicke Gravitational Field by Cerenkov Radiation

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Abstract

The possibility that a charged particle propagating in a gravitational field described by Brans-Dicke theory of gravity could emit Cerenkov radiation is explored. This process is kinematically allowed depending on parameters occurring in the theory. The Cerenkov effect disappears as the BD parameter $\omega \to \infty$, i.e. in the limit in which the Einstein theory is recovered, giving a signature to probe the validity of the Brans-Dicke theory.

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I. INTRODUCTION

Alternative theories of gravity [1] represent an extension of General Relativity in which the gravitational interaction is mediated by a tensor field and one or several scalar fields, in general massless. The Brans-Dicke (BD) theory of gravity [2] is, may be, the most famous prototype of such theories which incorparates Mach's principle and Dirac's large number hypothesis (see, for example, [3]) by means of a nonminimal coupling between the geometry and a scalar field ϕ, the BD scalar. The scalar field rules dynamics together with geometry and induces a variation of the gravitational *coupling* with time and space through the relation $G_{eff} = 1/\phi$. The Newtonian gravitational constant G_N is recovered in the limit $\phi \to constant$. As recent experiments [4,5] have shown, a variation of the Newton constant on astrophysical and cosmological sizes and time scale seems to have been confirmed.

During last years there has been a growing interest for the Brans-Dicke gravity, as well as for the alternative theories of gravity [1,6], due to the following reasons: *i*) The presence of scalar fields seems to be unavoidable in supersting theory [7]. *ii*) BD gravity follows from Kaluza-Klein theory in which the compactified extra dimensions is essential for generating scalar fields [8]. *iii*) BD theory is an important ingredient in the scenario of extended and hyperextended inflation [9–12]. Besides, constraints of the coupling constant of BD theory are derived in relation to the observations of gravitational waves from compact binaries [13], whereas investigations concerning the collapse to black holes in the framework of BD has been carried out in [14]. Recently, consequences of the BD gravitational field on neutrino oscillations and Sagnac effect have been analyzed in [15] and [16], respectively. In this paper we suggest a new physical effect which involves the Cerenkov emission by charged particles propagating in curved space-time.

II. THE BD THEORY OF GRAVITY

The effective action describing the interaction of the scalar field ϕ nonminimally coupled with the geometry and the ordinary matter is given by [2]

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[\phi R - \omega\frac{\partial_\mu\phi\partial^\mu\phi}{\phi} + \frac{16\pi}{c^4}\mathcal{L}_m\right] , \tag{2.1}$$

where R is the scalar curvature, $\mathcal{L}_m$ is the matter contribution in the total Lagrangian density. The constant ω can be positive or negative. In what follows we will assume $\omega > 0$. On an experimental setting, ω is determined by observations and its value can be constrained by classical tests of General Relativity: the light deflection, the relativistic perihelion rotation of Mercury, and the time delay experiment, resulting in reasonable agreement with all available observations thus far provided $\omega \geq 500$ [1,17]. The most recent bounds on ω comes from Very-Long Baseline Radio Interferometry (VLBI) experiment and it is given by $\omega > 3000$ [18]. On the other hand, bounds on the anisotropy of the cosmic microwave background radiation (CMB) give the upper limit $\omega \leq 30$ [9]. Nevertheless, it must be mentioned that results obtained in Refs. [19], in which the BD theory during the radiation matter equality has been investigated, seem to indicate that at the moment there is no observable effect appearing from CMB. It is important to point out that Einstein's theory is recovered as

$\omega \to \infty$. In this limit, the BD theory becomes indistinguishable from General Relativity in all its predictions (this is not always true, see for example [20–22]).

Variation with respect to the metric tensor $g_{\mu\nu}$ and the scalar field ϕ yield the field equations [2]

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = \frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right) + \frac{1}{\phi}\left(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right) + \frac{8\pi}{c^4\phi}\, T_{\mu\nu} \tag{2.2}$$

for the geometric part, and

$$\frac{2\omega}{\phi}\,\Box\phi - \frac{\omega}{\phi^2}\phi_{,\mu}\phi^{,\mu} + R = 0 \tag{2.3}$$

for the scalar field. $\Box$ is the usual d'Alembert operator in curved space–time and $T_{\mu\nu}$ is the momentum–energy tensor of matter.

The line element describing a static and isotropic geometry is expressed as

$$ds^2 = -e^{2\alpha} dt^2 + e^{2\beta}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]\,, \tag{2.4}$$

where the functions α and β depend on the radial coordinate r. The general solution in the vacuum is given by [2]

$$e^{2\alpha} = e^{2\alpha_0}\left[\frac{1 - B/r}{1 + B/r}\right]^{2/\lambda}\,, \tag{2.5}$$

$$e^{2\beta} = e^{2\beta_0}\left(1 + \frac{B}{r}\right)^4 \left[\frac{1 - B/r}{1 + B/r}\right]^{2(\lambda - C - 1)/\lambda}\,, \tag{2.6}$$

$$\phi = \phi_0\left[\frac{1 - B/r}{1 + B/r}\right]^{-C/\lambda}\,, \tag{2.7}$$

where C, λ, ϕ_0, α_0 and β_0 are arbitrary constants. λ is related to the constant C through the relation

$$\lambda^2 = 1 + C + C^2\left(1 + \frac{\omega}{2}\right)\,, \tag{2.8}$$

and B is related to the mass M of the source. Without to lose of the generality, we assume $\alpha_0 = 0$ and $\beta_0 = 0$. Here $\omega > 3/2$ [2].

In the weak field approximation, the components of the metric tensor, $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$, and the BD scalar reduce to the form [2]

$$g_{00} \simeq -1 + \frac{2M\phi_0^{-1}}{c^2 r}\frac{4 + 2\omega}{3 + 2\omega}\,, \tag{2.9}$$

$$g_{ii} \sim 1 + \frac{2M\phi_0^{-1}}{c^2 r}\frac{2 + 2\omega}{3 + 2\omega}\,, \quad i = 1, 2, 3, \tag{2.10}$$

$$g_{0i} = 0\,, \qquad g_{ij} = 0\,, \quad i \neq j\,, \tag{2.11}$$

$$\phi = \phi_0 + \frac{2M}{c^2 r}\frac{1}{3 + 2\omega}\,. \tag{2.12}$$

In order to have the agreement with observations, Eqs. (2.9)-(2.12) are obtained by means of an appropriate choice of the constant entering in Eqs. (2.5)-(2.7) [2]

$$\lambda = \sqrt{\frac{2\omega+3}{2(\omega+2)}}\,, \quad C \cong -\frac{1}{2+\omega}\,, \quad \alpha_0 = 0 = \beta_0\,, \tag{2.13}$$

$$\phi_0 = \frac{4+2\omega}{G_N(3+2\omega)}\,, \quad B = \frac{M}{2c^2\phi_0}\sqrt{\frac{2\omega+4}{2\omega+3}}\,.$$

In the limit $\omega \to \infty$, the usual weak field approximation of General Relativity is recovered.

III. THE CERENKOV EFFECT

We now investigate the possibility that a charge particle, propagating in a gravitational field described by BD theory, could emit radiation through the Cerenkov process. Such a process has been analyzed some years ago by Gasperini [23]. Here we follow a different approach proposed in a recent paper by Gupta, Mohanty and Samal [24]: The Cerenkov process occurs owing to a different coupling of fermions and photons with the gravitational background. They show that the gravitational field acts as an effective refractive index, whose expression is given by [24]

$$n_T^2(k_0) = 1 - \frac{R^i{}_i}{|\eta^{00}|k_0^2} + \frac{k_i k^j}{2\mathbf{k}^2}\frac{R^i{}_j}{k_0^2|\eta^{00}|}\,, \tag{3.1}$$

for the transverse modes, and

$$n_L^2(k_0) = 1 - \frac{k_i k^j}{\mathbf{k}^2|\eta^{00}|}\frac{R^i{}_j}{k_0^2}\,, \tag{3.2}$$

for the longitudinal modes. $R^i{}_i$ is understood as the sum over the spatial indices of the Ricci tensor $R^\mu{}_\nu$, i.e. $R^i{}_i \equiv \sum_{i=1}^3 R^i{}_i$, k_0 is the frequency of the emitted photon γ ($k^\mu = (k^0, \mathbf{k})$, with $\mathbf{k} = (k^1, k^2, k^3)$), and η^{00} is the 00-component of the metric tensor in the inertial frame, $\eta_{\mu\nu} = (-1, 1, 1, 1)$.

The scattering process $f(p) \to f(p') + \gamma(k)$, responsible of the Cerenkov radiation, is analyzed in the local inertial frame of the incoming fermion $f(p)$ with momentum p, while $f(p')$ and $\gamma(k)$ are the outcoming fermion with momentum p' and the emitted photon with momentum k. As one can immediately realize, the Cerenkov emission is not vanishing in the inertial frame since the refractive index turns out to be proportional to the Ricci tensor [24].

The energy radiated via Cerenkov effect by a charged particle moving in a background gravitational field is given by

$$\frac{dE}{dt} = \frac{Q^2\alpha_{em}}{4\pi p_0^2}\int_{k_{01}}^{k_{02}} dk_0 \left[p_0(p_0-k_0) - \frac{1}{2}k_0^2\right] k_0 \frac{n_\gamma^2(k_0)-1}{n_\gamma^2(k_0)}\,, \tag{3.3}$$

where Q is the charge of the fermion emitting the photon, α_{em} is the electromagnetic coupling constant, and p_0 is the energy of the fermion. Here n_γ indicates the refractive index for photons.

The non-vanishing components of the Ricci tensor corresponding to the metric (2.4) are

$$R_{00} = -\frac{4B^2Cr^4}{(r^2-B^2)^4\lambda^2}\left(\frac{r-B}{r+B}\right)^{2(2+C)/\lambda}, \tag{3.4}$$

$$R_{11} = R_{rr} = -\frac{4B[B^2C\lambda + Cr^2\lambda + Br(2+C-2\lambda^2)]}{r(r^2-B^2)^2\lambda^2}, \tag{3.5}$$

$$R_{22} = R_{\theta\theta} = \frac{2BCr[-2B(1+C)r + B^2\lambda + r^2\lambda]}{(r^2-B^2)^2\lambda^2}, \tag{3.6}$$

$$R_{33} = R_{\varphi\varphi} = \frac{2BCr[-2B(1+C)r + B^2\lambda + r^2\lambda]\sin^2\theta}{(r^2-B^2)^2\lambda^2}, \tag{3.7}$$

whereas the scalar curvature is

$$R = \frac{4B^2r^4}{(r^2-B^2)^4\lambda^2}\left(\frac{r-B}{r+B}\right)^{2(1+C)/\lambda} C^2\omega\,. \tag{3.8}$$

The summation over the spatial components of Ricci's tensor is then

$$R^i_i = \frac{4B^2r^4}{(r^2-B^2)^4\lambda^2}\left(\frac{r-B}{r+B}\right)^{2(1+C)/\lambda} C(-1+C\omega)\,. \tag{3.9}$$

The crucial point in order for the Cerenkov radiation to be kinematically allowed is that the refractive index (Eqs. (3.1) and/or (3.2)) is greater than 1. To show that this occurs in BD theory, we shall discuss some particular cases.

- If the momentum of photons is directed as $\mathbf{k} = (k, 0, 0)$, and $r \gg B$, then the last term in (3.1) and (3.2) reads

$$\frac{k_i k^j}{\mathbf{k}^2}\frac{R^i_j}{k_0^2} \sim \frac{R_{11}}{k_0^2} \sim -\frac{4BC}{r^3\lambda k_0^2} \sim \frac{4M}{r^3c^2\phi_0 k_0^2}\frac{1}{2\omega+3}, \tag{3.10}$$

where Eqs. (2.13) have been used. Similarly, one gets

$$R^i_i \sim \frac{8M^2}{r^4c^4\phi_0^2}\frac{\omega+1}{(2\omega+3)^2}\,. \tag{3.11}$$

Therefore, Eq. (3.1) assumes the form

$$n_T^2 \sim 1 + \frac{4M}{r^3c^2\phi_0 k_0^2}\frac{1}{2\omega+3} - \frac{8M^2}{r^4c^4\phi_0^2 k_0^2}\frac{\omega+1}{(2\omega+3)^2}, \tag{3.12}$$

which is greater than 1 being last term of the order $\mathcal{O}(r^{-4})$. The longitudinal refractive index turns out to be lesser than 1, as follows from (3.2).

- As before observed, B is a constant related to the source mass, and C is an arbitrary constant entering in the BD field equations describing the gravitational field (Eqs. (2.5)-(2.7)). The weak field solutions, Eqs. (2.9)-(2.12), are permissible *only* if the gravitational source is a suitable mass distribution, as for example the Sun, which generates everywhere a small field, inside and outside to it (see Eq. (3.12)). This does not hold for a point mass source or, in the more general case, for high density of matter [2]. Let us consider the gravitational field generated by a source placed at the center of Galaxies. In such a case, the field weak approximation cannot be trivially used, at least in the BD conjecture, because of the strong field regime inside the gravitational source. This situation avoids of using the constraints (2.13). Thus, as follows from (3.1), (3.2) and (3.9), $n_T^2 > 1$ provided R_i^i is negative and the last term in (3.1) and (3.2) is positive for $\omega > 0$, $C > 0$ and for r sufficiently large with respect to B. The condition $R_i^i < 0$ implies the constraint $\omega C < 1$. As expected, the effect disappears for $C = 0$, i.e. when the static solution (2.5)-(2.6) reduces to the solution of Einstein's theory.

- One can also investigate the case $\lambda \gg 1$ ($\omega \gg 1$) and $r \gg B$. Eqs. (3.5)-(3.7) and (3.9) becomes

$$R_{11} \sim -\frac{B}{r^3\sqrt{\omega}} \quad R_{22} \sim \frac{B}{r\sqrt{\omega}}\,, \quad R_{33} \sim \frac{B\sin\theta}{r\sqrt{\omega}}\,, \quad R_i^i \sim \frac{B^2}{r^4}\,.$$

  Notice these expressions do not depend on C. For photons propagating with momentum $\mathbf{k} = (k, 0, 0)$ the refractive indices are given by

$$n_T^2 \sim 1 + \frac{B}{r^3 k_0^2\sqrt{\omega}} - \frac{B^2}{r^4 k_0^2} > 1\,, \quad n_L^2 \sim 1 - \frac{B}{r^3 k_0^2\sqrt{\omega}} < 1\,.$$

The above results clearly show that the BD theory of gravity is a suitable framework for the occurrence of the Cerenkov effect, showing up a departure from General Relativity. Nevertheless, it must be noted that large bounds on ω yield a refractive index of the order $n_\gamma^2 \sim \mathcal{O}(1)$. Thus, from an experimental setting, the Cerenkov process is very difficult to be detected.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have analyzed, in the framework of BD theory of gravity, the Cerenkov emission by a charged particle. Since the Cerenkov radiation occurs for particles with very high energy, the suitable context for producing it could be represented by astrophysical and cosmological scenarios. For example, one can envisage the studying of astrophysical objects present at the center of Galaxies. Let us recall that it is widely believed that the central mass of Galaxies has to be a black hole described by General Relativity. But in our own galaxy, for instance, if Sgr A* (the super-massive compact object name) is a black hole, its luminosity should be three order of magnitude bigger than that which is observed. This discrepancy is called *the blackness problem*. In addition, observational data come from regions at a radius

larger than 4×10^4 Schwarzschild radius of a black hole of mass 2.6×10^6 solar masses -which is the inferred mass of the central object-, and so, proofs of the existence of a super-massive black hole in the center of the galaxy are not *conclusive* (alternative models, in fact, have been proposed in [25,26]). It is then of current interest to edit the analysis of a super-massive star placed at the center of galaxies whose gravitational field could satisfy the requirement of Mach's principle and look for Cerenkov's radiation emitted by accelerated charged particles, which is an allowed process in the BD theory of gravity, but vanishes in the framework of General Relativity. As a consequence, the detection of the Cerenkov radiation might give a strong signature in favour of BD theory.

It would be interesting to extend this analysis to scalar tensor-theories theories of gravity (as well as higher order theories of gravity), in which the strength of the coupling between the scalar field and gravity is determined by the function $\omega(\phi)$. In the more general case, a self–interaction potential $V(\phi)$, which plays a non trivial role on the dynamics of the field, can be also introduced. The dependence of the parameter ω on ϕ could have the property that, at the present epoch, the value of the scalar field ϕ_0 is such that ω is very large, leading to theories almost identical to General Relativity today, but for past or future values of ϕ, ω could take values that would lead to significant differences from General Relativity, hence to an enhancement of Cerenkov effect. Such extension is currently under investigation.

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